

# Lorentz-invariance violating effects in the Bose-Einstein condensation of an ideal bosonic gas

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We have studied the effects of Lorentz-invariance violation in the Bose-Einstein condensation (BEC) of an ideal bosonic gas, assessing both the nonrelativistic and ultrarelativistic limits. Our model describes a massive complex scalar field coupled to a CPT-even and Lorentz-violating background. First, by starting from the nonrelativistic limit of our model and by using experimental data, we give upper limits for some parameters of our model. But, the existence of the nonrelativistic BEC, in a Lorentz-invariance violating (LIV) framework, imposes strong restrictions on some LIV parameters. It is shown that only the critical temperature gains LIV contributions. In the sequel, we analyze the ultrarelativistic Bose-Einstein condensation, constructing a well-defined partition function for the relativistic bosonic ideal gas, from which severe constraints on certain LIV parameters are imposed. The analysis of the ultrarelativistic BEC has shown that the critical temperature and the critical chemical potential are slightly affected by LIV contributions.

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## I. INTRODUCTION

The CPT- and Lorentz-symmetry violations have been intensively investigated in the latest years. A strong motivation to study the CPT- and Lorentz-symmetry breaking is the necessity to get some information about underlying physics at Planck scale where the Lorentz symmetry may be broken due to quantum gravity effects, possibility opened up in early 90's [1]. Another reason is the need of examining the limits of validity of the CPT theorem and the Lorentz symmetry, based on the search for small deviations from scenarios characterized by CPT and Lorentz symmetry exactness. This line of investigation is conducted mainly in two contexts, one within the framework of the Standard Model Extension (SME) [1, 2] and another into the framework of the Planck scale modified dispersion relations [3]. The SME incorporates terms governing the effects of the spontaneous symmetry breaking of the CPT- and Lorentz-invariance in all sectors of the Standard Model of the fundamental interactions and particles. The main researches are devoted to the study of LIV effects in classical and quantum electrodynamics with the objectives to establish strong upper limits over the parameters ruling the CPT- and Lorentz-invariance violating effects. In this sense, a set of many investigations and diverse experimental setups, based on distinct theories and effects, have been proposed to constrain the Lorentz-violating parameters leading to the upper limits presented in Ref. [4].

The study of LIV effects in statistical physics into the context of the SME has been initiated in Ref.[5], based on the maximum entropy approach. There, it was then considered a general nonrelativistic Hamiltonian, containing the Lorentz-violating terms coming from the SME fermion sector [6]. It was shown that the usual laws of thermodynamics remain unaffected and that the relevant corrections appear at the form of rotationally invariant functions of the LIV parameters. The theoretical framework developed in Ref. [5] was by first used to analyze the influence of Lorentz violation on Bose-Einstein condensation in Ref. [7], where it was demonstrated that the Lorentz-violating terms can change the shape and the phase of the ground-state condensate produced by means of trapping techniques. Tough, it was also mentioned that these modifications could unlikely serve to state good bounds on the LIV parameters. An alternative study of BEC in the LIV framework was recently performed in Ref. [8] via the use of deformed dispersion relation in statistical physics. Moreover, the LIV effects in other thermodynamical systems, as the electromagnetic sector of the SME, have been examined in Refs. [9–11] starting from a finite-temperature-field-theory approach. Specifically, it was studied the influence of the Lorentz-violating CPT-odd and CPT-even terms on the black body radiation and the anisotropies induced in the angular energy density distribution.

In this work, we discuss some Lorentz-violating effects on a bosonic system, described by a complex scalar field, able to support the Bose-Einstein condensation [12]. It is important to remark that the Bose-Einstein condensation

can open up a new route for searching for small deviations of Lorentz symmetry if refined and accurate experimental techniques are used. Therefore, Bose-Einstein condensation could provide a new set of laboratory tests relevant for restricting Lorentz-violation parameters even more. The aim of the present work is to study the effects of the Lorentz-invariance violation in Bose-Einstein's condensation of an ideal bosonic gas both in the nonrelativistic and ultrarelativistic limits via the finite-temperature-field-theory formalism. In section II, we present the model that will be used to describe both the nonrelativistic and ultrarelativistic BEC. In section II.A, we take the nonrelativistic limit of our model with the following purposes. First, to study the nonrelativistic BEC and, second, to establish upper bounds for some of the LIV parameters. The existence of the BEC phenomenon in LIV backgrounds imply strong restrictions over some of our LIV coefficients which are related with some parameters of the SME. In section II.B, we study the charged relativistic bosonic gas. The construction of a well-defined partition function implies strong constraints on one LIV parameter. In section III, we give our final remarks and conclusions.

## II. A CPT-EVEN AND LORENTZ-INVARIANCE VIOLATING MODEL FOR THE COMPLEX SCALAR FIELD

The simplest Lorentz-invariance violating Lagrangian for the complex scalar field in (1+3)-dimensions is

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\kappa^\mu (\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*) + \lambda^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi - m^2 \phi^* \phi, \quad (1)$$

where  $\kappa^\mu$  is a CPT-odd Lorentz-violating four-vector with mass dimension 1 and,  $\lambda^{\mu\nu}$  is a dimensionless symmetric tensor ruling the CPT-even and Lorentz-invariance violating contributions. The second term in Lagrangian (1) was already used to study Lorentz-violating effects on topological defects generated by scalar fields in (1+1) dimensions [13]. A similar term has been also adopted to study the influence of Lorentz violation on the relativistic version of acoustic black holes generated in an Abelian Higgs model [14]. Further, in the context of the aether-like models, the model (1) can be considered as an extension of those obtained in Ref. [15].

However, it is worthwhile to observe that the CPT-odd LIV term ruled by  $\kappa^\mu$  can be eliminated by an appropriate canonical field redefinition

$$\phi \rightarrow e^{i\hat{\kappa} \cdot x} \varphi, \quad \phi^* \rightarrow e^{-i\hat{\kappa} \cdot x} \varphi^*, \quad (2)$$

with  $\hat{\kappa}^\mu$  chosen as  $\hat{\kappa}^\mu = (g^{\mu\nu} + \lambda^{\mu\nu})^{-1} \kappa_\nu$ . Note that the inverse of the expression in parentheses does exist because the Lorentz-violating parameter  $\lambda^{\mu\nu}$  is small compared to 1. By expressing the Lagrangian (1) in terms of the new field  $\varphi$ , we get

$$\mathcal{L} \rightarrow \partial_\mu \varphi^* \partial^\mu \varphi + \lambda^{\mu\nu} \partial_\mu \varphi^* \partial_\nu \varphi - (m^2 + \hat{\kappa}_\mu \kappa^\mu) \varphi^* \varphi, \quad (3)$$

which no longer exhibits the CPT-violating term depending on  $\kappa^\mu$ , whereas the mass term acquires a small correction. If the term  $i\kappa^\mu (\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*)$  can be removed with such a canonical transformation, it is not a true LIV term and the  $\kappa^\mu$  vector cannot be measurable. Taking into account these considerations, we set  $\kappa^\mu = 0$ . Therefore, from now on we will only consider the CPT-even and Lorentz-violating term in the lagrangian density for the complex scalar field, that is

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + \lambda^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi - m^2 \phi^* \phi. \quad (4)$$

### A. A nonrelativistic Bose-Einstein condensation in a CPT-even and Lorentz-violating framework

For applications of the model (4) in low energy situations, we compute its associated nonrelativistic lagrangian density,

$$\mathcal{L}' = i(1 + \lambda_{00}) \psi^* \partial_t \psi - \psi^* \mathcal{H}'_C \psi, \quad (5)$$

where  $\mathcal{H}'_C$  is the canonical hamiltonian density given by

$$\mathcal{H}'_C = -\frac{1}{2m} \nabla^2 + \frac{1}{2m} \lambda_{jk} \partial_j \partial_k + i\lambda_{0j} \partial_j + \frac{1}{2} \lambda_{00} m. \quad (6)$$

The modified Schrödinger equation generated by the nonrelativistic Lagrangian density is

$$-i(1 + \lambda_{00})\partial_t\psi + \mathcal{H}'_C\psi = 0. \quad (7)$$

The first term in the previous equation plays the role of the Planck's constant ( $\hbar$ ), in such a way that  $\lambda_{00}$  may be limited by the relative uncertainty ( $\Delta\hbar/\hbar$ ). From this, one writes

$$\lambda_{00} \leq 3.6 \times 10^{-8}, \quad (8)$$

where  $3.6 \times 10^{-8}$  is the best value for the ratio  $\Delta\hbar/\hbar$  provided by CODATA 2006 [16].

If we perform the following field rescaling:  $\psi \rightarrow (1 + \lambda_{00})^{-1/2}\psi$  and  $\psi^* \rightarrow (1 + \lambda_{00})^{-1/2}\psi^*$ , the modified Schrödinger equation would be read as

$$-i\partial_t\psi - \frac{1}{2m(1 + \lambda_{00})}\nabla^2 + \frac{1}{(1 + \lambda_{00})}\left(\frac{1}{2m}\lambda_{jk}\partial_j\partial_k + i\lambda_{0j}\partial_j + \frac{1}{2}\lambda_{00}m\right)\psi = 0. \quad (9)$$

By looking the second term, we can set an upper limit for  $\lambda_{00}$  using the relative uncertainty of  $(\hbar^2/2m)$  that can be obtained using the respective relative uncertainties of  $(\hbar/m)$  and  $(\hbar)$ . For example, for the  $^{87}\text{Rb}$  atom we get  $\lambda_{00} \leq 4.9 \times 10^{-8}$ , and for the electron we obtain  $\lambda_{00} \leq 3.74 \times 10^{-8}$ . Both results are compatible with the value obtained in Eq.(8).

The lagrangian (5) is invariant under the following global field transformation:  $\psi \rightarrow e^{-i\alpha}\psi$ ,  $\psi^* \rightarrow e^{i\alpha}\psi^*$ , whose conserved charge density is  $(1 + \lambda_{00})\psi^*\psi$ . The conjugate momenta to  $\psi$  and  $\psi^*$  are  $\pi^* = i(1 + \lambda_{00})\psi^*$  and  $\pi = 0$ , respectively.

In the nonrelativistic regime, the partition function is

$$Z'(\beta) = \int \mathcal{D}\psi \mathcal{D}\psi^* \mathcal{D}\pi \mathcal{D}\pi^* \delta[\pi] \delta[\pi^* - i(1 + \lambda_{00})\psi^*] \exp \left\{ \int_{\beta} dx i\pi^* \partial_{\tau}\psi + i\pi \partial_{\tau}\psi^* - \mathcal{H}'_C + \mu(1 + \lambda_{00})\psi^*\psi \right\}, \quad (10)$$

where  $\mu$  is the chemical potential associated to the conserved charge  $(1 + \lambda_{00})\psi^*\psi$ . The integration is performed over the fields satisfying periodical boundary conditions in the  $\tau$  variable:  $\psi(\tau, \mathbf{x}) = \psi(\tau + \beta, \mathbf{x})$  and  $\psi^*(\tau, \mathbf{x}) = \psi^*(\tau + \beta, \mathbf{x})$ . By performing the momentum integrations and some integrations by parts, the partition function reads as

$$Z(\beta) = \int \mathcal{D}\psi \mathcal{D}\psi^* \exp \left\{ - \int_{\beta} dx \psi^* \mathbf{D} \psi \right\}, \quad (11)$$

where the operator  $\mathbf{D}$  is

$$\mathbf{D} = \partial_{\tau} - \frac{1}{2m\gamma}\nabla^2 + \frac{1}{2m\gamma}\lambda_{jk}\partial_j\partial_k + \frac{i}{\gamma}\lambda_{0j}\partial_j - \left(\mu - \frac{\lambda_{00}m}{2\gamma}\right), \quad (12)$$

with  $\gamma = 1 + \lambda_{00}$ .

In absence of LIV interactions, the operator defined in Eq. (12) is  $(\partial_{\tau} - \frac{1}{2m}\nabla^2 - \mu)$ , whose zero-mode is intimately related with the existence of the Bose-Einstein condensation. It allows to guarantee that the BEC phenomenon occurs when  $\mu \rightarrow 0^-$ . The value  $\mu = 0$  is the fundamental value for occurring BEC. Such condition is similar to the superconductivity phase transition: it only happens when the value of the resistivity is zero [17].

Now, from Eq.(12), the existence of a zero-mode leads to the requirement  $\mu - \lambda_{00}m/2\gamma = 0$ . As the value  $\mu = 0$  is measured in the laboratory, the existence of the zero mode linked to BEC in a LIV framework implies that

$$\lambda_{00} = 0, \quad (13)$$

which is compatible with the limits obtained for it. Also, under the condition (13),  $\gamma = 1$ .

Therefore, by imposing restriction (13) and computing the functional integration (11) in the Fourier space, the partition function becomes

$$\ln Z(\beta) = -V \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_n \ln \left[ i\beta\omega_n + \frac{\beta}{2m}\mathbf{p} \cdot \mathbf{N}\mathbf{p} - \beta\lambda_{0j}p_j - \beta\mu \right], \quad (14)$$

with  $\omega_n$  being the bosonic Matsubara's frequencies,  $\omega_n = 2\pi n/\beta$ ,  $n = 0, \pm 1, \pm 2, \dots$ . We have defined  $\mathbb{N}$  as a symmetric matrix whose components are  $N_{ij} = \delta_{ij} - \lambda_{ij}$ . It will be positive-definite if the components  $\lambda_{jk}$  are sufficiently small. By performing the summation in Eq.(14), we get

$$\ln Z(\beta) = -V \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \ln \left[ 1 - \exp \left( \frac{\beta}{2m} \mathbf{p} \cdot \mathbb{N} \mathbf{p} - \beta \lambda_{0j} p_j - \beta \mu \right) \right]. \quad (15)$$

Now, we make the following operations under the momentum integral. First, we do a translation  $p_k \rightarrow p_k + (N^{-1})_{kj} \lambda_{0j}$ . Next we perform a rotation  $\mathbf{p} \rightarrow \mathbb{R} \mathbf{p}$ , such that  $\mathbb{R}$  diagonalizes the matrix  $\mathbb{N}$ , i. e.,  $\mathbb{R}^T \mathbb{N} \mathbb{R} = \mathbb{D}$ , where  $\mathbb{D}$  is a diagonal matrix whose elements are the eigenvalues of  $\mathbb{N}$ . Finally, we make the following rescaling  $\mathbf{p} \rightarrow \mathbb{D}^{-1/2} \mathbf{p}$ . Under such manipulations the partition function (15) becomes

$$\ln Z(\beta) = -V (\det \mathbb{N})^{-1/2} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \ln \left[ 1 - e^{-\beta(\epsilon_1 - \mu)} \right], \quad (16)$$

where  $\epsilon_1 = \mathbf{p}^2/2m - m (N^{-1})_{kj} \lambda_{0k} \lambda_{0j}/2$ . The quantity  $m (N^{-1})_{kj} \lambda_{0k} \lambda_{0j}/2$  is positive-definite. The nonrelativistic BEC, in an ideal bosonic gas, must occurs when  $\mu \rightarrow 0^-$ , therefore, it implies that  $\lambda_{0k}$  must be null:

$$\lambda_{0j} = 0. \quad (17)$$

Therefore the partition function becomes

$$\ln Z(\beta) = -V (\det \mathbb{N})^{-1/2} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \ln \left[ 1 - e^{-\beta(\epsilon - \mu)} \right], \quad (18)$$

with  $\epsilon = \mathbf{p}^2/2m$ . The physical considerations yielding conditions (13) and (17) impose that only purely spacelike CPT-even parameters ( $\lambda_{ij}$ ) contribute for the nonrelativistic partition function. The contribution of the terms  $\lambda_{ij}$  was also observed in the results of Refs.[5, 7]. We observe that in Eq. (18) the full LIV contributions are contained in the factor  $(\det \mathbb{N})^{-1/2}$ , consequently, by setting as null the LIV parameters, i. e.,  $N_{ij} = \delta_{ij}$ , we obtain the partition function for the complex scalar field in absence of Lorentz-violating backgrounds.

The nonrelativistic particle density is given by

$$\rho = \frac{1}{2\pi^2} (\det \mathbb{N})^{-1/2} \int_0^\infty dp \frac{p^2}{e^{\beta(\epsilon - \mu)} - 1}. \quad (19)$$

Here, the chemical potential must be negative ( $\mu < 0$ ), once the particle density is non-negative. Note that Eq.(19) is an implicit formula for  $\mu$  as a function of  $\rho$  and  $T$ . For  $T$  above some critical temperature  $T_c$ , one can always find one value of  $\mu$  for which Eq. (19) holds. If the density  $\rho$  is held fixed and the temperature is lowered,  $\mu \rightarrow 0^-$ , in the region  $T \geq T_c$  one achieves

$$\rho = \left( \frac{m}{2\pi\beta} \right)^{3/2} \zeta(3/2) (\det \mathbb{N})^{-1/2}, \quad (20)$$

while the critical temperature is

$$T_c = T^{(BEC)} (\det \mathbb{N})^{1/3} = T^{(BEC)} \left[ 1 - \frac{1}{3} \text{tr}(\lambda_{ij}) + \dots \right]. \quad (21)$$

Here,  $T^{(BEC)} = 2\pi m^{-1} [\rho/\zeta(3/2)]^{2/3}$  is the critical temperature in absence of LIV interactions. A expansion in (21) can be used to establish an upper limit for the parameter  $\text{tr}(\lambda_{ij})$  by using the relative uncertainty of the BEC temperature. With this purpose, the BEC temperature  $T^{(BEC)}$  is experimentally determined in the range 0.5–2  $\mu\text{K}$  [17]. However, the most refined experiments are able to obtain temperatures of the order of  $0.5 \times 10^{-10} \text{K}$  [19]. If we consider  $10^{-11}$  as the lowest temperature ever detected, the relative uncertainty for BEC temperature would be  $2 \times 10^{-5} - 5 \times 10^{-6}$ . Thus, we establish the following upper-bound

$$\text{tr}(\lambda_{ij}) < 3 \times 10^{-6}. \quad (22)$$

At temperatures  $T < T_c$ , expression (20) becomes an equation for charge density  $\rho - \rho_0$  of the nonzero momentum ( $\mathbf{p} \neq \mathbf{0}$ ) states,

$$\rho - \rho_0 = \left( \frac{m}{2\pi\beta} \right)^{3/2} \zeta(3/2) (\det \mathbb{N})^{-1/2}, \quad (23)$$

so that the charge density in the ground state ( $\mathbf{p} = \mathbf{0}$ ) is

$$\rho_0 = \rho \left[ 1 - \left( \frac{T}{T_c} \right)^{3/2} \right]. \quad (24)$$

This shows that the fraction of the condensate density is not modified by the Lorentz-violating interactions. The condition for nonrelativistic BEC,  $\rho \ll m^3$ , is maintained.

The bonafide description of the nonrelativistic BEC in LIV backgrounds by our scalar model allows to construct in a consistent way the partition function of a relativistic charged ideal gas and to analyze the ultrarelativistic Bose-Einstein condensation in a LIV framework.

### B. The relativistic ideal gas in a CPT-even and Lorentz-invariance violating framework

The model of Lagrangian (4) is invariant under the  $U(1)$  global symmetry,  $\phi \rightarrow e^{-i\alpha}\phi$  and  $\phi^* \rightarrow e^{i\alpha}\phi^*$ , where  $\alpha$  is any real constant. The conserved charge density, expressed in terms of the fields and their conjugate momenta, is:  $j^0 = i\phi^*\pi - i\phi\pi^*$ . It preserves the same canonical structure of the Lorentz invariant case. The canonical conjugate momenta associated to  $\phi$  and  $\phi^*$  are  $\pi^* = \dot{\phi}^*(1 + \lambda_{00}) - \lambda_{0j}\partial_j\phi^*$  and  $\pi = \dot{\phi}(1 + \lambda_{00}) - \lambda_{0j}\partial_j\phi$ , respectively. This model does not possess constraints and its canonical Hamiltonian density,

$$\mathcal{H}_C = (1 + \lambda_{00})^{-1} [\pi^*\pi + \pi^*\lambda_{0j}\partial_j\phi + \pi\lambda_{0j}\partial_j\phi^* + \lambda_{0j}\lambda_{0j}\partial_j\phi^*\partial_j\phi] + \nabla\phi^* \cdot \nabla\phi + m^2\phi^*\phi - \lambda_{jk}\partial_j\phi^*\partial_k\phi, \quad (25)$$

is positive-definite for  $\lambda^{\mu\nu}$  sufficiently small. The partition function is defined as

$$Z(\beta) = \int \mathcal{D}\phi \mathcal{D}\phi^* \mathcal{D}\pi \mathcal{D}\pi^* \exp \left\{ \int_{\beta} dx [i\pi^*\partial_{\tau}\phi + i\pi\partial_{\tau}\phi^* - \mathcal{H}_C + \mu j^0] \right\}, \quad (26)$$

where  $\mu$  is the chemical potential. The functional integration is performed over the fields satisfying periodic boundary conditions,  $\phi(\tau + \beta, \mathbf{x}) = \phi(\tau, \mathbf{x})$ , and  $\phi^*(\tau + \beta, \mathbf{x}) = \phi^*(\tau, \mathbf{x})$ . By performing the momentum integrations and some integrations by parts in the sequel, the partition function takes the form

$$Z(\beta) = \int \mathcal{D}\phi \mathcal{D}\phi^* \exp \left\{ - \int_{\beta} dx \phi^* \mathbf{D}_R \phi \right\}, \quad (27)$$

where

$$\mathbf{D}_R = -\eta(\partial_{\tau} - \mu)^2 + 2\lambda_{\tau k}\partial_k(\partial_{\tau} - \mu) - (\delta_{jk} - \lambda_{jk})\partial_j\partial_k + m^2, \quad (28)$$

and we have made the following definitions:  $\eta = 1 - \lambda_{\tau\tau}$ ,  $\lambda_{\tau\tau} = -\lambda_{00}$  and  $\lambda_{\tau j} = -i\lambda_{0j}$ . The functional integration in (27) is computed in the momentum space, yielding

$$\ln Z(\beta) = -V \int \frac{d\mathbf{p}}{(2\pi)^2} \sum_n \ln [\beta^2 \tilde{\mathbf{D}}_R(n, \mathbf{p})], \quad (29)$$

$$\tilde{\mathbf{D}}_R(n, \mathbf{p}) = \eta(\omega_n + i\mu)^2 - 2\lambda_{\tau j}p_j(\omega_n + i\mu) + (\delta_{jk} - \lambda_{jk})p_jp_k + m^2, \quad (30)$$

where  $\omega_n$  the bosonic Matsubara's frequencies,  $\omega_n = \frac{2\pi n}{\beta}$ ,  $n = 0, \pm 1, \pm 2, \dots$

Performing the summation in (29),

$$\begin{aligned} \sum_n \ln \left[ a (2\pi n + i\beta\mu)^2 - 2b (2\pi n + i\beta\mu) + c^2 \right] &= \ln \left[ 1 - \exp \left( -\frac{1}{a} \sqrt{ac^2 - b^2} + \beta\mu + i\frac{b}{a} \right) \right] \\ &+ \ln \left[ 1 - \exp \left( -\frac{1}{a} \sqrt{ac^2 - b^2} - \beta\mu - i\frac{b}{a} \right) \right]. \end{aligned} \quad (31)$$

where we have ruled out irrelevant constants and, the parameters  $a, b, c$  are given by

$$a = \eta, \quad b = \beta \lambda_{\tau j} p_j, \quad c^2 = \beta^2 [(\delta_{jk} - \lambda_{jk}) p_j p_k + m^2]. \quad (32)$$

We observe that in Eq. (31) the chemical potential gains an imaginary part,  $\eta^{-1} \beta \lambda_{\tau j} p_j$ , which changes the bosonic character of the field. The possibility of statistical transmutation depending on the linear momentum and the temperature does not happen in physical systems. Therefore, to describe a relativistic bosonic ideal gas in a CPT-even and Lorentz-violating framework we must impose that the coefficients  $\lambda_{\tau k}$  are null:

$$\lambda_{\tau k} = 0. \quad (33)$$

Under such physical requirement, Eq.(31) becomes simpler

$$\sum_n \ln \left[ a (2\pi n + i\mu)^2 + c^2 \right] = \ln \left[ 1 - \exp \left( -\frac{c}{\sqrt{a}} + \mu \right) \right] + \ln \left[ 1 - \exp \left( -\frac{c}{\sqrt{a}} - \mu \right) \right], \quad (34)$$

and the partition function will describe a relativistic bosonic ideal gas composite of charged particles.

Thus, after performing the summation, the partition function (29) becomes

$$\begin{aligned} \ln \mathcal{Z} &= -V \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \ln \left[ 1 - \exp \left( -\beta \eta^{-1/2} \sqrt{\mathbf{p} \cdot \mathbb{N} \mathbf{p} + m^2} + \beta \mu \right) \right] \\ &- V \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \ln \left[ 1 - \exp \left( -\beta \eta^{-1/2} \sqrt{\mathbf{p} \cdot \mathbb{N} \mathbf{p} + m^2} - \beta \mu \right) \right], \end{aligned} \quad (35)$$

where we have defined  $\mathbb{N}$  as a symmetric matrix, whose elements are  $N_{jk} = \delta_{jk} - \lambda_{jk}$ , and that is positive-definite for sufficiently small values of  $\lambda_{jk}$ . Now, we take the following operations under the momentum integral. First, we perform a rotation  $\mathbf{p} \rightarrow \mathbb{R} \mathbf{p}$ , such that  $\mathbb{R}$  diagonalizes the matrix  $\mathbb{N}$ , i. e.,  $\mathbb{R}^T \mathbb{N} \mathbb{R} = \mathbb{D}$ , where  $\mathbb{D}$  is a diagonal matrix whose elements are the eigenvalues of  $\mathbb{N}$ . Finally, we make the rescaling  $\mathbf{p} \rightarrow \eta^{1/2} \mathbb{D}^{-1/2} \mathbf{p}$ . The partition function (35) is then rewritten as

$$\ln \mathcal{Z} = -V \eta^{3/2} (\det \mathbb{N})^{-1/2} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left\{ \ln [1 - \exp(-\beta \omega + \beta \mu)] + \ln [1 - \exp(-\beta \omega - \beta \mu)] \right\}, \quad (36)$$

with  $\omega = \sqrt{\mathbf{p}^2 + M^2}$  and  $M^2 = \eta^{-1} m^2$ . Both integrals converge if  $|\mu| \leq M$  and the ultrarelativistic BEC occurs when  $\mu = \pm M$ . We observe that in Eq. (36) the full LIV contributions are contained in the factor  $(\det \mathbb{N})^{-1/2}$  and in  $\eta^{-1}$ . Consequently, by setting as null the LIV parameters, i.e.,  $N_{ij} = \delta_{ij}$ ,  $\eta = 1$ , we recover the partition function for the relativistic complex scalar field in absence of Lorentz-violating backgrounds [18].

### 1. The CPT-even and Lorentz-invariance violating contribution to the ultrarelativistic BEC

We follow Ref.[18] to describe the ultrarelativistic BEC in this CPT-even and Lorentz-violating framework. Thus, for  $|\mu| < M$  the charge density is

$$\rho = \eta^{3/2} (\det \mathbb{N})^{-1/2} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left( \frac{1}{e^{\beta(\omega - \mu)} - 1} - \frac{1}{e^{\beta(\omega + \mu)} - 1} \right). \quad (37)$$

This equation is really an implicit formula for  $\mu$  as a function of  $\rho$  and  $T$ . For  $T$  above some critical temperature  $T_c$ , one can always find a value for  $\mu$  such that Eq. (37) holds. If the density  $\rho$  is maintained fixed and the temperature is lowered, the chemical potential  $\mu$  increases until the point  $|\mu| = M$  is reached. Thus, in the region  $T \geq T_c \gg M$ , we obtain

$$|\rho| \approx \frac{1}{3} M (\det \mathbb{N})^{-1/2} \eta^{3/2} T^2. \quad (38)$$

When  $|\mu| = M$  and the temperature is lowered even further such that  $T < T_c$ , the charge density is written as

$$\rho = \frac{1}{\beta V} \left( \frac{\partial \ln Z}{\partial \mu} \right)_{\mu=M} = \rho_0 + \rho^*(\beta, \mu = M), \quad (39)$$

where  $\rho_0$  is a charge contribution from the condensate (the zero-momentum mode) and the  $\rho^*(\beta, \mu = M)$  is the thermal particle excitations (finite-momentum modes) which is given by Eq. (37) with  $|\mu| = M$ .

The critical temperature  $T_c$ , in which the Bose-Einstein condensation occurs, is reached when  $|\mu| = M$ , and is determined implicitly by the equation  $\rho = \rho^*(\beta_c, \mu = M)$ , so that

$$T_c = T^{(BEC)} (\det \mathbb{N})^{1/4} \eta^{-1/2} = T^{(BEC)} \left[ 1 + \frac{1}{2} \lambda_{\tau\tau} - \frac{1}{4} \text{tr}(\lambda_{ij}) + \dots \right], \quad (40)$$

with  $T^{(BEC)} = (3|\rho|/m)^{1/2}$  being the BEC critical temperature [18] in absence of the Lorentz-invariance violation. Observe that the CPT-even coefficients give first-order contributions.

At temperatures  $T < T_c$ , expression (39) is an equation for the charge density  $\rho - \rho_0$  of the nonzero momentum ( $p \neq 0$ ) states,

$$\rho - \rho_0 = \frac{1}{3} M (\det \mathbb{N})^{-1/2} \eta^{3/2} T^2, \quad (41)$$

so that the charge density in the ground state ( $p = 0$ ) is

$$\rho_0 = \rho \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]. \quad (42)$$

Thus, differently from the critical temperature and the chemical potential, the fraction of the condensate charge is not modified by the Lorentz-violating interactions. In absence of the LIV terms, the necessary condition for an ideal Bose gas of mass  $m$  to undergo Bose-Einstein condensation at a ultrarelativistic temperature ( $T_c \gg m$ ) is that  $\rho \gg m^3$ . Such a condition is slightly modified by the LIV interactions to  $\rho \gg (1 - \lambda_{\tau\tau})^{-3/2} m^3$ .

### III. REMARKS AND CONCLUSIONS

We have analyzed both nonrelativistic and ultrarelativistic Bose-Einstein condensation of an ideal Bose gas via a simple model for a complex scalar field containing CPT-even and Lorentz-violating terms. Initially, we have studied the nonrelativistic case whose partition function is well-defined. The existence of the Bose-Einstein condensation phenomenon in this framework imposes strong restrictions on the Lorentz-violating parameters,  $\lambda_{0j} = 0$  and  $\lambda_{00} = 0$ . This implies that only the pure space-like coefficients,  $\lambda_{ij}$ , contributes to the nonrelativistic BEC of an ideal bosonic gas in a LIV framework. In addition, by using the experimental data for BEC's temperatures, we achieve the following upper-bound,  $\text{tr}(\lambda_{ij}) < 3 \times 10^{-6}$ . It is worth to observe that the full LIV contributions are contained in the factor  $(\det \mathbb{N})$ , thus, in absence of LIV backgrounds,  $N_{ij} = \delta_{ij}$ , the usual nonrelativistic BEC's phenomena can be reproduced. In this regime, the condensate charge density fraction remains unaltered and just the critical temperature is corrected by LIV contributions.

The appropriate description of the nonrelativistic BEC has suggested the possibility to analyze, in a consistent way, the ultrarelativistic Bose-Einstein condensation of a charged relativistic ideal bosonic gas in this LIV framework. In this case, for obtaining a well-defined relativistic partition function describing the correct statistical behavior of

charged bosons, we must impose strong constraints on the LIV parameters,  $\lambda_{\tau j} = 0$ . The remaining LIV coefficients affect the critical temperature and the chemical potential, while the condensate charge density remains unaltered. It is worth to notice that the full LIV contributions are contained in the factors  $\det \mathbb{N}$  and  $\eta$  thus, in absence of LIV backgrounds, i.e.  $N_{ij} = \delta_{ij}$  and  $\eta = 1$ , the usual ultrarelativistic BEC's phenomena can be obtained.

Therefore, we can affirm that the existence of BEC's phenomenon, in both nonrelativistic and ultrarelativistic cases, imposes that the LIV parameters  $\lambda_{0j}$  (related to  $\lambda_{0\tau}$ ) are null. The parameter  $\lambda_{00}$  (related to  $\lambda_{\tau\tau}$ ) is not constrained in the relativistic case but **is set as** null in the nonrelativistic limit. On the other hand, the parameters  $\lambda_{ij}$  give contributions in both limits.

It is worthwhile to observe that all the LIV parameters of (4) present in the nonrelativistic Lagrangian density (5) can be related with some parameters of the SME determined by the Lagrangian of free spin-1/2 Dirac fermions of mass  $m$  [2, 5]. Specifically, our scalar model reproduces the non spinor terms of the nonrelativistic limit of the free fermionic Lorentz-violating Hamiltonian, given by Eq. (13) of Ref. [5], and the Hamiltonian (1) of Ref. [7]. We justify such connection by remembering that, in absence of LIV backgrounds, the form of the Schrodinger equation for the description of nonrelativistic bosonic free systems is universal depending only on the particle mass. Similarly, it is the form of Pauli's equation for the free spin-1/2 particle. By eliminating the spinorial terms from free Pauli's equation, the remaining ones constitute the Schrodinger equation for free bosonic particles. A similar association can be established in the presence of Lorentz-violating backgrounds. It is worth to notice that, the magnitude of the LIV coefficients depends in the peculiar properties of each particle. Thus we do not transfer the experimental-bounds from the scalar sector to the fermion one. In true, we are proposing a strategy that opens the possibility to establish certain limits in some LIV coefficients of the fermionic sector without considering the spin of the particle. This situation is similar to case when we study the hydrogen atom without considering the contribution of electron spin. Based on those statements, we can thus establish the following correspondence:

$$\frac{1}{2}\lambda_{00}m \rightarrow a_0 - mc_{00} - me_0, \quad (43)$$

$$m\lambda_{0j} \rightarrow a_j - m(c_{0j} + c_{j0}) - me_j, \quad (44)$$

$$\lambda_{ij} \rightarrow 2c_{ij} + c_{00}\delta_{ij} \quad (45)$$

By considering an exact correspondence and using our restrictions and limits, we can establish that

$$a_0 - mc_{00} - me_0 = 0, \quad (46)$$

$$a_j - m(c_{0j} + c_{j0}) - me_j = 0, \quad (47)$$

$$\frac{2}{3}\text{tr}(c_{ij}) + c_{00} < 10^{-6}. \quad (48)$$

The first and second constraints on the fermionic SME parameters are still not reported in the literature [4]. The third equation gives a good upper bound, but there are even better ones in the literature [4].

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- [1] V. A. Kostelecky and S. Samuel, Phys. Rev. Lett. **63**, 224 (1989); Phys. Rev. Lett. **66**, 1811 (1991); Phys. Rev. D **39**, 683 (1989); Phys. Rev. D **40**, 1886 (1989), V. A. Kostelecky and R. Potting, Nucl. Phys. B **359**, 545 (1991); Phys. Lett. B **381**, 89 (1996); V. A. Kostelecky and R. Potting, Phys. Rev. D **51**, 3923 (1995).  
[2] D. Colladay and V. A. Kostelecky, Phys. Rev. D **55**, 6760 (1997); D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998).



- [3] T. Jacobson, S. Liberati, D. Mattingly, *Annals Phys.* **321**, 150 (2006); T. Jacobson, S. Liberati, D. Mattingly, *Phys.Rev. D* **67**, 124011 (2003); T. Jacobson, S. Liberati, D. Mattingly, *Nature* **424**, 1019 (2003); T. Jacobson, S. Liberati, D. Mattingly, *Phys.Rev. D* **67**, 124011 (2003).
- [4] V. Alan Kostelecky and Neil Russell, *Data Tables for Lorentz and CPT Violation*, arXiv:0801.0287 [hep-ph].
- [5] D. Colladay and P. McDonald, *Phys. Rev. D* **70**, 125007 (2004).
- [6] V.A. Kostelecky and C. D. Lane, *J. Math. Phys.* **40**, 6245 (1999); V. A. Kostelecky and R. Lehnert, *Phys. Rev. D* **63**, 065008 (2001); D. Colladay and V. A. Kostelecky, *Phys. Lett. B* **511**, 209 (2001); O. G. Kharlanov and V. Ch. Zhukovsky, *J. Math. Phys.* **48**, 092302 (2007); R. Lehnert, *Phys. Rev. D* **68**, 085003 (2003); R. Lehnert, *J. Math. Phys.* **45**, 3399 (2004); G. M. Shore, *Nucl. Phys. B* **717**, 86 (2005); W. F. Chen and G. Kunstatter, *Phys. Rev. D* **62**, 105029 (2000).
- [7] D. Colladay and P. McDonald, *Phys. Rev. D* **73**, 105006 (2006).
- [8] E. Castellanos and A. Camacho, *Mod. Phys. Lett. A* **25**, 459 (2010).
- [9] R. Casana, M. M. Ferreira Jr. and J. S. Rodrigues, *Phys. Rev. D* **78**, 125013 (2008); J. M. Fonseca, A. H. Gomes, W. A. Moura-Melo, *Phys. Lett. B* **671**, 280 (2009).
- [10] R. Casana, M. M. Ferreira Jr., J. S. Rodrigues, Madson R.O. Silva, *Phys. Rev. D* **80**, 085026 (2009).
- [11] R. Casana, M. M. Ferreira, Jr, M. R.O. Silva, *Phys.Rev. D* **81**, 105015 (2010).
- [12] C. J. Pethick and H. Smith, *Bose-Einstein condensation in dilute gases*, Cambridge University Press, 2002; A. Griffin, T. Nikuni and E. Zaremba, *Bose-Condensed Gases at Finite Temperatures*, Cambridge University Press, 2009; L. Pitaevskii and S. Stringari, *Bose- Einstein Condensation*, Oxford University Press, 2003.
- [13] D. Bazeia, M. M. Ferreira Jr., A. R. Gomes and R. Menezes, *Physica D* **239**, 942 (2010).
- [14] M. A. Anacleto, F. A. Brito, E. Passos, *Phys. Lett. B* **694**, 149 (2010).
- [15] M. Gomes, J. R. Nascimento, A. Yu. Petrov and A. J. da Silva, *Phys. Rev. D* **81**, 045018 (2010).
- [16] P. J. Mohr, B. N. Taylor, and D. B. Newell, *Rev. Mod. Phys.* **80**, 633 (2008)..
- [17] J. F. Annett, *Superconductivity, Superfluids, and Condensates*, Oxford University Press, 2009.
- [18] H. E. Haber and H. A. Weldon, *Phys. Rev. Lett.* **46**, 1497 (1981); H. E. Haber and H. A. Weldon, *Phys. Rev. D* **25**, 502 (1982).
- [19] D. M. Weld, P. Medley, H. Miyake, D. Hucul, D. E. Pritchard, and W. Ketterle, *Phys. Rev. Lett.* **103**, 245301 (2009).